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SUGGESTED EXPRESSION FOR ASYMMETRIC MEASUREMENT UNCERTAINTY INTERVALS

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Background

- Uncertainty intervals are typically reported as symmetric intervals around the measured value
- Symmetric interval can not be used for high expanded uncertainties (e.g. U = 100 %) (this is not covered well in documents like GUM and Eurachem uncertainty guideline).
- In many instrumental techniques results are generated by multiplicative combination of random variables, i.e. distributions of results are driven towards a log-normal distribution (i.e. an asymmetric distribution)
- At small or modest relative standard uncertainties (<15 to 20 %) normal and log-normal distributions are so similar that the normal distribution can serve as a suitable approximation
- At large relative standard uncertainties (>15 to 20 %) asymmetry should be considered.

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Large uncertainties (u_c > 15 to 20 %)

A common way of handling this is by transforming data using $\log_{10} x$ or $\log_e x$

Uncertainty intervals for a result x: $\frac{x}{10^{k \times s_{loge}}}$ to $x \times 10^{k \times s_{loge}}$

or

$\frac{x}{{}^F U}$ to $x \times {}^F U$

where ${}^F U$ is the "uncertainty factor" calculated as $10^{k \times s_{loge}}$

It is typically assumed that distribution of measurement results can be approximated with either a normal distribution or a log-normal distribution

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Suggested alternative approach for handling asymmetry in measurement results

Transformation using

$$x_{trans} = x^B$$

where B is a parameter that is optimized with the goal that transformed data should have a skewness close to 0, i.e. become symmetric

$$x_{trans} = x^{B_{opt}}$$

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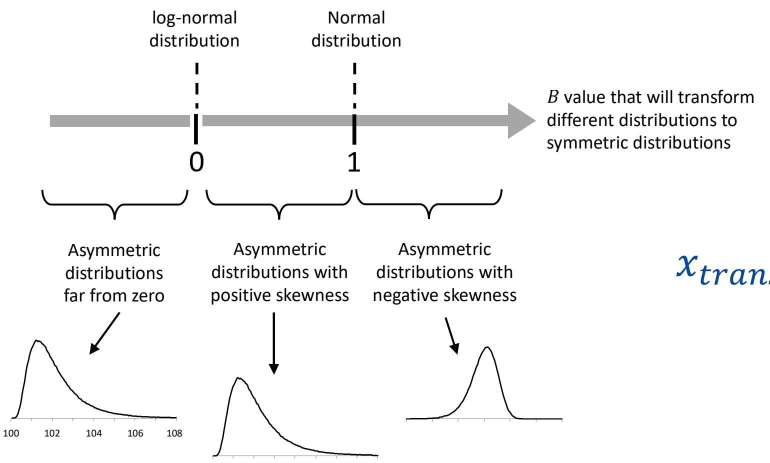
Suggested alternative approach for handling asymmetry in measurement results



$$x_{trans} = x^{B_{opt}}$$

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Suggested alternative approach for handling asymmetry in measurement results

Finding B_{opt}:

- ❑ Using mathematical tools available in many calculation softwares
- ❑ In reality it is difficult to find B_{opt} for the population based on experimental data. Large numbers of data (typically >10^3 to 10^4) are needed that are rarely available
- ❑ Control samples and control charts describing within-laboratory reproducibility can contain data in the order of 10^2 and in rare cases 10^3.

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Suggested alternative approach for handling asymmetry

It is suggested that without any other information of a proper value for B_{opt}

- ❑ assume B_{opt} equal to 1 when CV < 15 % (i.e. no transformation of the data is performed)
- ❑ for these low CV the value of B_{opt} is not critical (different values of B_{opt} will result in similar uncertainty intervals)
- ❑ for CV > approx. 15 to 20 % it is often sensible to assume B_{opt} close to 0 (for instance 0.0001), i.e. to assume a log-normal distribution
- ❑ a proper value of B_{opt} might also be obtained from Monte Carlo simulations if a relevant model equation is available
- ❑ alternatively, a general agreed value of B_{opt} might be used

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Suggested alternative approach for handling asymmetry when the relative standard uncertainty is independent of measurand level

Two sources of asymmetry in the uncertainty intervals

- ❑ Asymmetry in the distribution of measurement results – handled by

  $x_{trans} = x^{B_{opt}}$

- ❑ Standard uncertainty will increase proportional to the measurand – handled by

  $\frac{x}{1+k\times u_{rel}}$ to $\frac{x}{1-k\times u_{rel}}$

  where k is the coverage factor and u_{rel} is the relative standard uncertainty

Combination of these

$$\frac{x}{(1+k\times u_{rel,trans})^{1/B_{opt}}} \quad \text{to} \quad \frac{x}{(1-k\times u_{rel,trans})^{1/B_{opt}}}$$

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Suggested alternative approach for handling asymmetry when the relative standard uncertainty is independent of measurand level

$$\frac{x}{(1+k\times u_{rel,trans})^{1/B_{opt}}} \text{ to } \frac{x}{(1-k\times u_{rel,trans})^{1/B_{opt}}}$$

B_{opt} approaching 0
i.e. when measurement results approaches a log-normal distribution

$$\frac{x}{10^{k\times u_{log10}}} \text{ to } x\times 10^{k\times u_{log10}}$$

B_{opt} approaching 1
i.e. when measurement results approaches a normal distribution

$$\frac{x}{1+k\times u_{rel}} \text{ to } \frac{x}{1-k\times u_{rel}}$$

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Suggested alternative approach for handling asymmetry

General issue when measurement results have an asymmetric distribution:

- Transformation based on $\log_{10} x$, $\log_e x$ and x^{Bopt} will result in symmetry around the median
- The uncertainty interval in the original space will cover the median with a given probability (95 % when using k equal to 1.96)

Is the median equal to what is intended to be measured?

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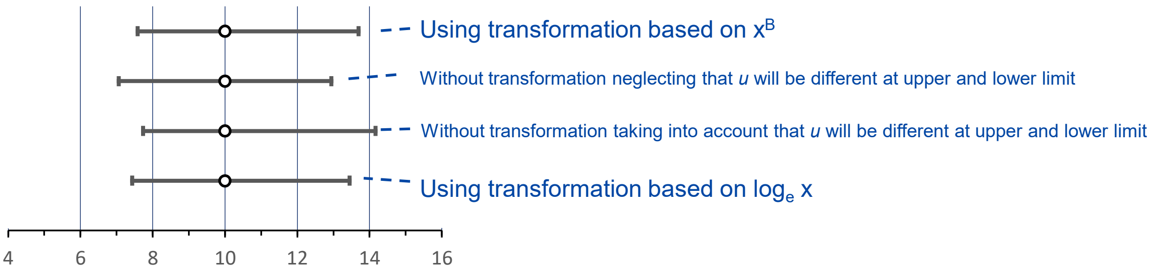
Example - Comparison of different ways of expressing uncertainty intervals (95 %) (when the relative uncertainty is independent of concentration)

Determination of sulfur in gas samples using gas chromatography and chemiluminescence detection
From control sample (n=740): CV_{Rw}=15 % and $B_{opt} \approx 0.4$



At CV_{Rw} of 15 to 20 % this starts to be of importance. At larger CV_{Rw} this will be more pronounced.

More information in E. Sahlin, B. Magnusson, Expression for uncertainty intervals handling skewness when the relative standard uncertainty is independent of the measurand level, Accreditation and Quality Assurance, 27 (2022) 223-233.

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The end!

Thank you for listening

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