The Use of Measurement Uncertainty in an Operating Mine

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Mine Activities Using Test Results and Associated Measurement Uncertainties

- Exploration to expand reserves.
- Breaking up deposit material (drilling and blasting rocks).
- Deciding which is ore and which waste.
- Separating mineral(s) of interest from gangue (the concentration process).
- Shipping product to the customer.
Decision Points in the Process

- Does the blasted material go to the mill or to waste?
- How much metal was fed from mine to mill?
- Are the mineral separations efficient?
- Does the mass of metal in the concentrate plus that in the tailings match what was in the mill feed?
- What is the value of the metal in the product being sold?

WHAT ROLE DOES MEASUREMENT UNCERTAINTY PLAY?
A Look at Mine Statistics

- The Cu grade of the ore is 1.17%.
- The cut-off grade (is it waste or does it go to the mill?) depends on metal prices.
- An example is a cut-off grade of 0.10% Cu. Anything less than 0.10% goes to waste at a low Cu price but not necessarily at a higher one.

What Goes to Waste?

- With Cu at $3.00/lb (LME price in June 2010) $6.62 worth of Cu is in 1000 kg of 0.10% Cu rock.
- With Cu at $4.25/lb (LME price April 30, 2011) $9.37 worth of Cu is in 1000 kg of rock running 0.10% Cu.
- If it costs $7.00 to treat 1000 kg of rock through the mill, then 0.10% Cu material would go to waste in the first case and to the mill in the second.
But What of Measurement Uncertainty?

- If the expanded (k=2) uncertainty of the sampling and analysis of Cu in the rock at 0.1% Cu is 0.01%, the amount of Cu could vary from 0.9 kg to 1.1 kg in 1000 kg.

- The value of Cu in the rock at $3.00/ lb ranges from $5.94 to $7.26 per 1000 kg.

- Given these numbers the decision would probably be to send the rock to waste.

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But What of Measurement Uncertainty?

- If the expanded uncertainty of the sampling and analysis of Cu in the rock at 0.1% Cu is 0.02%, the Cu in 1000 kg would range from 0.8 kg Cu to 1.2 kg Cu.

- The value of Cu in the rock at $3.00/ lb ranges from $5.28 to $7.92 per 1000 kg.

- Given these numbers, the decision of waste vs. mill is more difficult to make.

- The larger uncertainty makes the decision less clear cut.
But What of Measurement Uncertainty?

- At $4.25/lb, the value of Cu in 1000 kg of rock with an expanded uncertainty of 0.01% would lie in the range $8.42 to $10.20.
- At $4.25/lb, the value of Cu in 1000 kg of rock with an expanded uncertainty of 0.02% would lie in the range $7.48 to $11.22.
- Thus with the higher Cu value there is no doubt about the rock going to the mill.

Decisions, Decisions, Decisions

- The mine operators will have to establish decision rules to deal with situations such as the $5.28 - $7.92 range of Cu values seen in an earlier slide.
- The fact that they know the measurement uncertainty gives them a basis for making an informed decision.
- The measurement uncertainty must include sampling and sub-sampling variances.
The Mine/Mill Mass Balance is a regular calculation done to monitor a mining operation in an attempt to balance the amount of metal going into a mill against the amount coming out of the mill in the concentrate plus that in the mill tails.

This calculation involves three sampling events and three analyses:
- Mill feed
- Mill tails
- Concentrate

This means three different measurement uncertainties.
The three uncertainties each involve variability due to:

- Sampling and sub-sampling (lack of homogeneity)
- Analysis

Sampling the mill feed is usually the largest source of variability because the material sampled is of large particle size. Sampling in the mill can be significant.

<table>
<thead>
<tr>
<th>Range % Cu</th>
<th>Standard Deviation Calculated from Duplicate Analyses in the Lab</th>
<th>Standard Deviation Calculated from Duplicate Mill Feed Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;0.01 to 0.10</td>
<td>0.00335 % Cu (relative)</td>
<td>0.00729 % Cu (relative)</td>
</tr>
<tr>
<td>&gt;0.10 to 1.0</td>
<td>0.0594 % Cu (relative)</td>
<td>0.136 % Cu (relative)</td>
</tr>
<tr>
<td>&gt; 1.0 to 10</td>
<td>0.0495 % Cu (relative)</td>
<td>0.351 % Cu (relative)</td>
</tr>
<tr>
<td>25% (Concentrate)</td>
<td>0.479 % Cu (relative)</td>
<td>N.A.</td>
</tr>
</tbody>
</table>
Assume the average grade of the 90,000 metric tons (90,000,000 kg) of ore going into the mill daily is 1.1 % Cu.

The standard deviation of the sampling of that mill feed is 0.136 % Cu (previous slide).

The uncertainty of how much Cu enters the mill daily when estimated using 2 SD is 990,000 ± 2700 kg Cu.

This is over $20,000 worth of Cu at $4.00/lb

If the Cu recovery is 95%, 5% goes to tailings and 95% goes to concentrates.

This means 940,500 kg of the Cu reports to the concentrate and 49500 kg to the tailings.

The concentration in the tailings would be 100 x 49,500/89,000,000 = 0.0556% Cu.

The SD at this concentration is 0.00335 % Cu as shown in the previous table.
### Mine/Mill Mass Balance

<table>
<thead>
<tr>
<th>Ore to mill kg</th>
<th>Cu in Concentrate kg</th>
<th>Cu in tails kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>90,000,000 (95% recovery of 1.1% Cu in ore)</td>
<td>940,500 kg</td>
<td>49,500 kg</td>
</tr>
<tr>
<td>Standard Deviation of Cu results absolute</td>
<td>0.00489 kg Cu</td>
<td>0.00335 kg Cu</td>
</tr>
<tr>
<td>MU of Cu results (2 SD's)</td>
<td>9200 kg</td>
<td>332 kg</td>
</tr>
</tbody>
</table>

The reconciliation of how much Cu went into the mill and how much came out is subject to large uncertainties.

1. ± 2700 kg Cu going into the mill.
2. ±9200 kg Cu in the concentrate.
3. ±332 kg Cu in the mill tailings.

These large uncertainties mean that getting a perfect reconciliation of the mass balance is difficult.
Measurement Uncertainty for Cu Concentrate Shipment Analyses

- The analysis of concentrate shipments is critical because of the large dollar values of such shipments.
- For this reason it is important to get a reliable estimate of measurement uncertainty in that analysis.

An important first step is to identify the likely sources of variability contributing to the uncertainty.

There are three sources:
1. Uncertainty in the value quoted for the certified reference material \( V_{RM} \).
2. Uncertainty in the analysis \( V_b \).
3. Uncertainty in sampling during the loading \( V_{samp} \).
Measurement Uncertainty for Cu Concentrate Shipment Analyses

- $V_T = V_{RM} + V_b + V_{samp} = 0.00320 + 0.000625 + 0.00437 = 0.00879$
- $SD_T = (0.00879)^{1/2} = 0.0905 \% \text{ Cu}$
- Expanded uncertainty $U = 0.18 \% \text{ Cu}$
- The reported result in the 25 \% Cu concentration range is xx.xx +/- 0.18 \% Cu at a 95 \% level of confidence.

Concentrate Shipment Protocol

- The seller samples the concentrate as it is loaded onto the ship.
- The seller analyses the samples using rigorous quality control.
- The buyer pays 90\% of the value of the shipment as determined by the seller's sampling and analyses.
- The buyer samples the shipment as it is unloaded and analyses the samples.
Concentrate Shipment Protocol

The two sets of results are compared and if they are within the pre-agreed splitting limits the final 10% of the payment is made based on the average of the two sets of results.

If they are not in agreement each party sends a cut of their sample to an agreed umpire laboratory.

Final payment is made based on the umpire laboratory results and the “losing” lab pays.

<table>
<thead>
<tr>
<th>ANALYTE</th>
<th>TOTAL SAMPLES</th>
<th>TOTAL SAMPLES (Without Umpire)</th>
<th>TOTAL SAMPLES TO UMPIRE</th>
<th>TOTAL SAMPLES TO UMPIRE (Won)</th>
<th>TOTAL SAMPLES TO UMPIRE (Lost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>2963</td>
<td>2770</td>
<td>193</td>
<td>87</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>93.49%</td>
<td>6.51%</td>
<td>45.08%</td>
<td>54.92%</td>
</tr>
<tr>
<td>Cooper</td>
<td>1862</td>
<td>1667</td>
<td>195</td>
<td>94</td>
<td>101</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>89.53%</td>
<td>10.47%</td>
<td>48.21%</td>
<td>51.79%</td>
</tr>
<tr>
<td>Zinc</td>
<td>986</td>
<td>930</td>
<td>56</td>
<td>35</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>94.32%</td>
<td>5.68%</td>
<td>62.50%</td>
<td>37.50%</td>
</tr>
</tbody>
</table>
Measurement Uncertainty in Shipment Settlements

- The splitting limits are set based on the negotiated agreement between the parties.
- Common splitting limits are set in the range 0.25 to 0.50% Cu by mutual agreement.
- The expanded MU of 0.18 % Cu quoted earlier for concentrate shipments allows the seller to be confident of being able to detect a bias of this magnitude.

What Role Does Measurement Uncertainty Play?

- Measurement uncertainty permits more knowledgeable decisions to be made:
  - Does material go to waste or to the mill?
  - Is the concentration process efficient?
  - Is the mass balance reconciliation acceptable?
- Is the sampling and analysis of concentration shipments good enough to detect a bias for negotiated splitting limit purposes?