GUM/Supplement 1 – Numerical Implementation

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Recent Development in Measurement Uncertainty
Lisbon 2011

Materials Science & Technology

MUSE – Measurement Uncertainty Simulation and Evaluation

PhD thesis of

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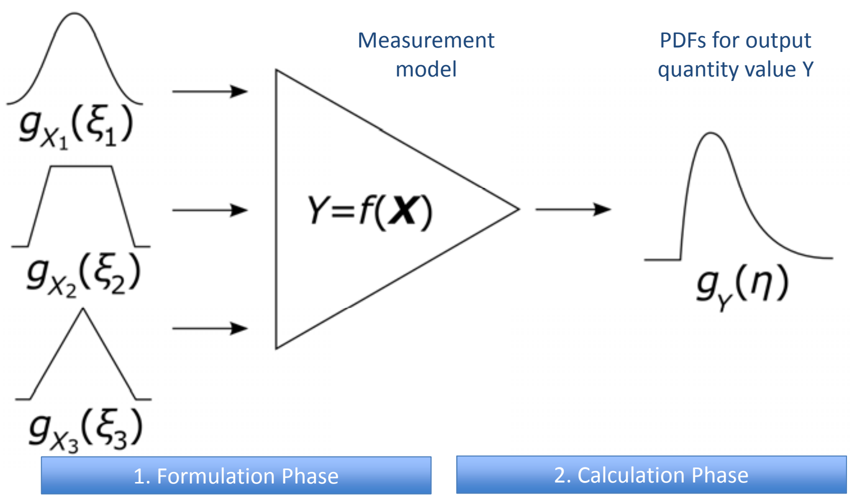
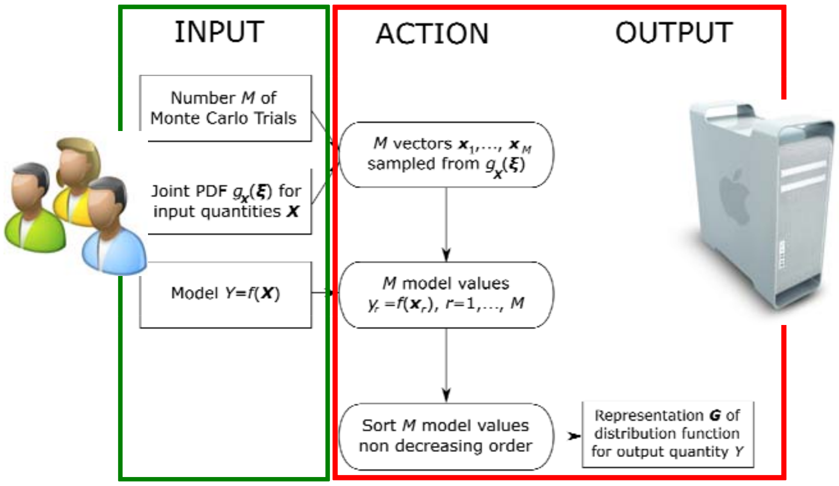
Swiss Federal Institute of Technology (ETH) Zürich

- Prof Walter Gander – Institute of Scientific Computing

Basic Principles

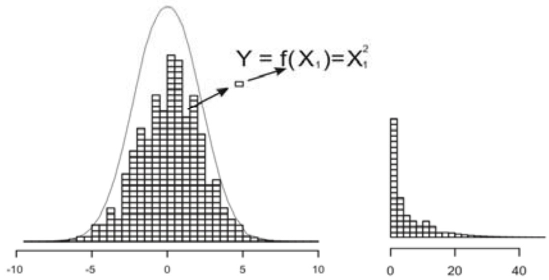


Basic Principles



Monte Carlo Method

- Sample of the input quantities
- Evaluate the given model for the given values
- Determines an approximation for the PDF of the output quantities



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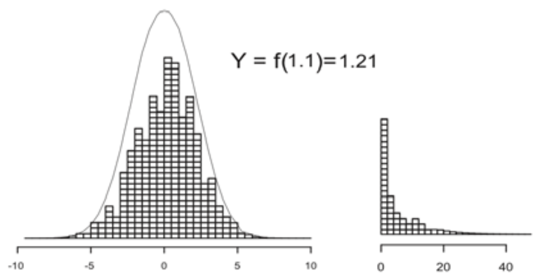

Monte Carlo Method

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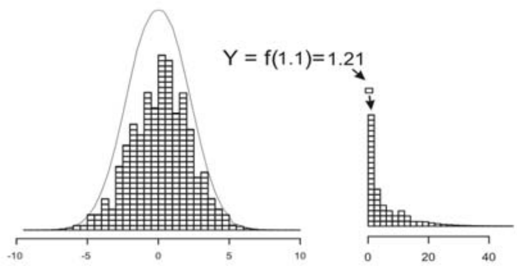


Monte Carlo Method

Random number generation for the input quantities

- Generation of random numbers of a rectangular distribution U(0, 1)
- Use these random numbers to generate the random numbers of the PDF of an input quantity
- The more random numbers the “better” accuracy

➢ Basis for an efficient implementation is the efficient generation of uniformly distributed random numbers

Monte Carlo Method

Efficient generation of U(0,1)

- Not every random number generator is suited for MC (e.g. Excel, Microsoft Visual C# etc.)
- Wichman-Hill
  - Proposed by GS1
  - "easy" to implement
  - Period $\approx 10^{37}$
- Mersenne Twister
  - "State of the art"
  - Standard implementation available
  - Period $\approx 10^{6000}$

Monte Carlo Method

Efficient Evaluation of the Equation of the Measurand

- An evaluation of the model function is needed for each value of the PDFs of the input quantities
- Interpretation of variables/mathematical functions (Parse)
- Time consuming procedures
- ➢ Economizing of these procedures means gains in efficiency

Monte Carlo Method

Efficient Evaluation of the Equation of the Measurand

- *MUSE* uses "block-by-block" evaluation
  - Buffer method with 10^4 values per block
  - All functions are calculated within these blocks
  - Model function has only the be "parsed" once for each block
- Parse the model function in the so call Postfix notation (reverse polish notation)
  - No bracketing needed

Monte Carlo Method

Efficient Summarization of the Results

- *MUSE* generates "vast" files with possible values of the PDF of the measurand
  - 10^8 evaluations ≈ 800 MB (binary)
- GS1 demands to sort all values
- Sorting is very time consuming (especially with a large number of simulations)

Monte Carlo Method

Efficient Summarization of the Results

- Statistical values (e.g. mean, standard deviation) are calculated for each block (block-by-block)
- The overall statistical parameters are calculated as mean values of the parameters of the blocks
- Different to GS1!
  - Convergence has been proven
  - Optional overall sorting has been implemented

Modeling Concept

- GUM and GS1 describe methods to calculate the measurement uncertainty → calculator kernel
- GUM and GS1 do not provide a basic concept for the dealing with comprehensive and elaborate measurement scenarios
- *MUSE* supports the advanced user for building, organizing and comprehension of models
  - Stored as pure text files
  - Open and easily adjustable format
  - Strict language definition in XML

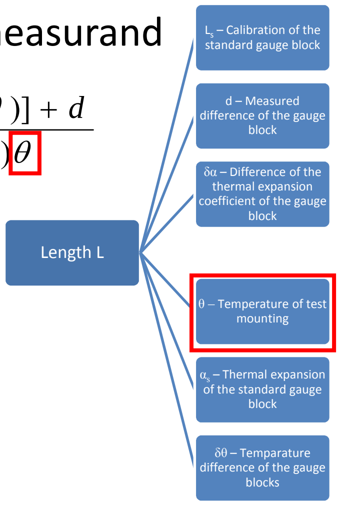
Modeling Concept

Example: Gauge block calibration

Modeling Concept

- Equation of the measurand

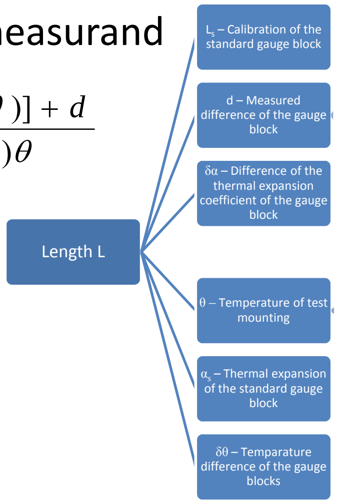
$$L = \frac{L_s[1 + \alpha_s(\theta - \delta\theta)] + d}{1 + (\alpha_s + \delta\alpha)\theta}$$



Modeling Concept

- Equation of the measurand

$$L = \frac{L_s[1 + \alpha_s(\theta - \delta\theta)] + d}{1 + (\alpha_s + \delta\alpha)\theta}$$



- Sub-influences

$$\theta = \boxed{\theta_0} + \Delta$$

$$d = D + d_1 + d_2$$

Modeling Concept

Influence quantities as distributions

```
<influence id="tq" name="Average temperature deviation">
  <distribution>
    <gauss>
      <mu>-0.1</mu>
      <sigma>0.2</sigma>
    </gauss>
  </distribution>
</influence>
```

$$\theta = \boxed{\theta_0} + \Delta$$

Modeling Concept

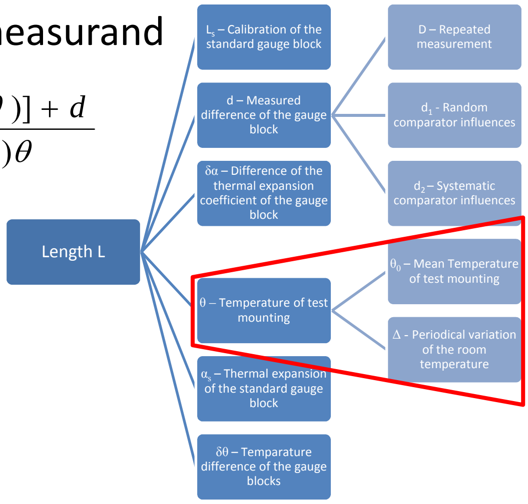
- Equation of the measurand

$$L = \frac{L_s[1+\alpha_s(\theta-\delta\theta)]+d}{1+(\alpha_s+\delta\alpha)\theta}$$

- Sub-influences

$$\theta = \theta_0 + \Delta$$

$$d = D + d_1 + d_2$$



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Modeling Concept

Modeling of complex influence quantities

```
<influence id="t" name="Deviation of temperature">
  <formula>tq + delta</formula>
  <influences>
    <influence id="tq" name="Average temperature deviation">
      <distribution>
        <gauss>
          <mu>-0.1</mu>
          <sigma>0.2</sigma>
        </gauss>
      </distribution>
    </influence>
    <influence id="delta" name="Effect of cyclic temperature variation">
      <distribution>
        <arcsine>
          <lower>-0.5</lower>
          <upper>0.5</upper>
        </arcsine>
      </distribution>
    </influence>
  </influences>
</influence>
```

$$\theta = \theta_0 + \Delta$$

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Modeling Concept

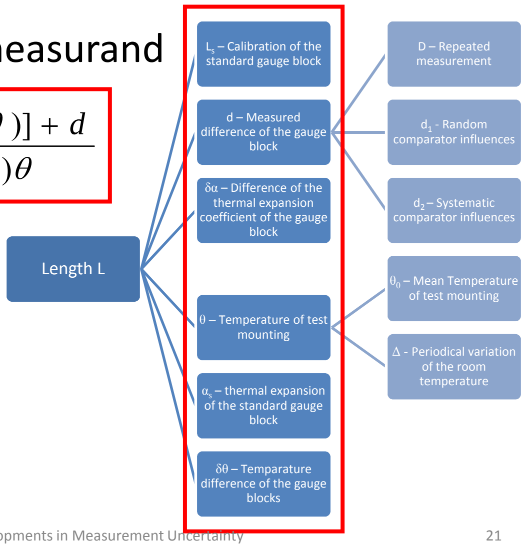
- Equation of the measurand

$$L = \frac{L_s[1 + \alpha_s(\theta - \delta\theta)] + d}{1 + (\alpha_s + \delta\alpha)\theta}$$

- Sub-influences

$$\theta = \theta_0 + \Delta$$

$$d = D + d_1 + d_2$$



Modeling Concept

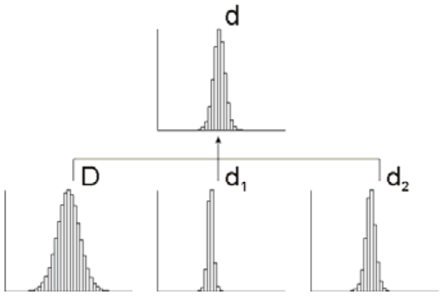
Complete model of gauge block calibration

```
<model name="length" targetid="l">
 <influence id="l" name="Length">
  <formula>(ls * (1 + as * (t - dt)) + d) / (1 + t * (as + da))</formula>

  <influences>
   <influence id="ls" name="Calibration of reference standard">
   <influence id="d" name="Length of the reference standard">
   <influence id="as" name="Thermal expansion coefficient">
   <influence id="t" name="Deviation of temperature">
   <influence id="da" name="Difference in expansion coefficient">
   <influence id="dt" name="Difference in temperatures">
  </influences>
 </influence>
</model>
```

$$L = \frac{L_s[1 + \alpha_s(\theta - \delta\theta)] + d}{1 + (\alpha_s + \delta\alpha)\theta}$$

Modeling Concept



$$d = D + d_1 + d_2$$

Modeling Concept

Operating System	Number of Evaluations
Windows Vista	10^6
	10^7
	10^8
Linux (openSUSE)	10^6
	10^7
	10^8

Beyond GUM-SP1

Use of *sensitivity coefficients* for the calculation of the effect of an individual influence quantity on the overall measurement uncertainty

1. Usage of quadratic terms:

$$w_{quad}(x_i) = \frac{c_i^2 u^2(x_i)}{\sum_{j=1}^{n} c_j^2 u^2(x_j)} = \frac{c_i^2 u^2(x_i)}{u^2(y)}$$

2. Usage of absolute terms:

$$w_{abs}(x_i) = \frac{|c_i| u(x_i)}{\sqrt{\sum_{j=1}^{n} c_j^2 u^2(x_j)}} = \frac{|c_i| u(x_i)}{u(y)}$$

Beyond GUM-SP1

1) Simulate
- All influence quantities generate random numbers
- Calculate *mean* and *standard deviation* for each influence quantity during the simulation

2) Simulate
- A part of the influence quantities are “locked” to the previously calculated mean value
(The effect of influence quantities are „*silenced*“)

Beyond GUM-SP1

Possible Scenarios:

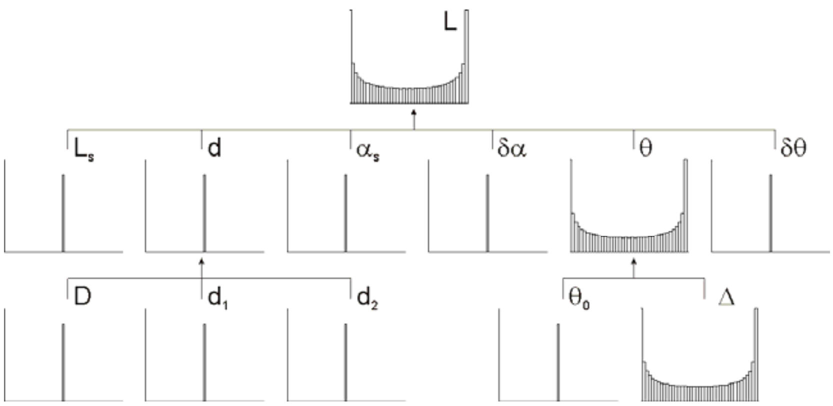
- Turn **on** only the investigated influence quantity
  - Only the investigated influence quantity generates random numbers

```
<simulation>
  <init>
    <instance name="length" model="length"/>
  </init>
  <calculation mcsimulations="100">
    <fadeoutlist mode="all" type="oneon"/>

    <analyse mode="on" histbars="40" datafiles="delete"/>
    <uncertainty>length</uncertainty>
  </calculation>
</simulation>
```


Beyond GUM-SP1

1. Option: Turn on only the investigated influence



Beyond GUM-SP1

Possible Scenarios:

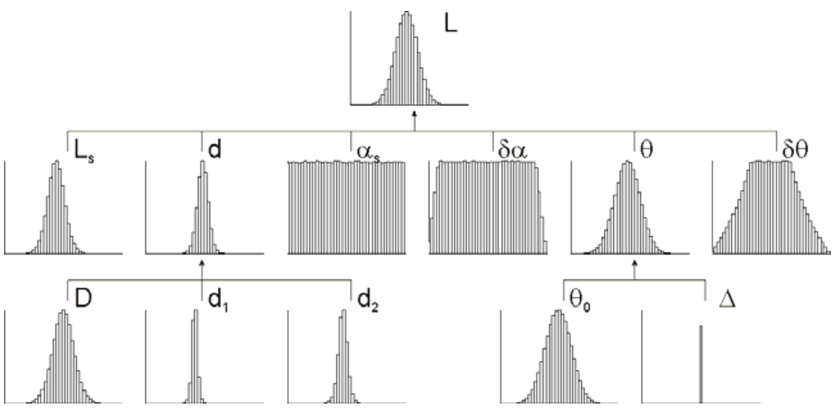
- Turn only the investigated influence quantity **off**
  - All others generate random numbers as beforehand

```
<simulation>
  <init>
    <instance name="length" model="length"/>
  </init>
  <calculation mcsimulations="100">
    <fadeoutlist mode="all" type="oneoff"/>

    <analyse mode="on" histbars="40" datafiles="delete"/>
    <uncertainty>length</uncertainty>
  </calculation>
</simulation>
```


Beyond GUM-SP1

2. Option: Turn only the investigated influence off



Beyond GUM-SP1

Classical GUM

- Quadratic terms

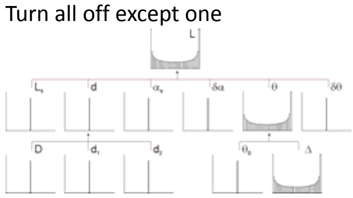
$$w_{quad}(x_i) = \frac{c_i^2 u^2(x_i)}{\sum_{j=1}^{n} c_j^2 u^2(x_j)} = \frac{c_i^2 u^2(x_i)}{u^2(y)}$$

- Absolut values

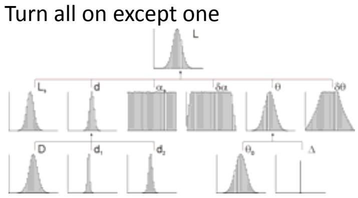
$$w_{abs}(x_i) = \frac{|c_i| u(x_i)}{\sqrt{\sum_{j=1}^{n} c_j^2 u^2(x_j)}} = \frac{|c_i| u(x_i)}{u(y)}$$

Monte Carlo Method

- Turn all off except one



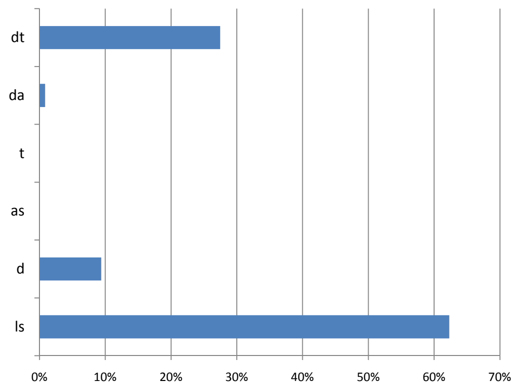
- Turn all on except one



Beyond GUM-SP1

- GUF with quadratic terms: $w_{quad}(x_i) = \frac{c_i^2 u^2(x_i)}{\sum_{j=1}^{n} c_j^2 u^2(x_j)} = \frac{c_i^2 u^2(x_i)}{u^2(y)}$

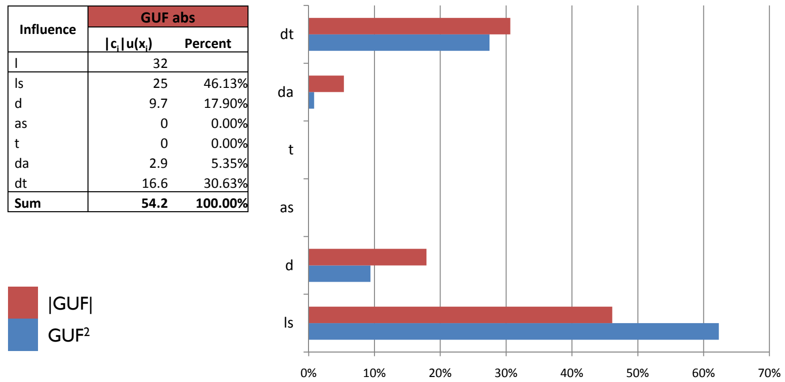
Influence	GUF Quad	
	$c_i^2u^2(x_i)$	Percent
l	1003	
ls	625	62.31%
d	94.09	9.38%
as	0	0.00%
t	0	0.00%
da	8.41	0.84%
dt	275.56	27.47%
Sum	**1003.06**	**100.00%**



Beyond GUM-SP1

- GUF with absolute values: $w_{abs}(x_i) = \frac{|c_i|u(x_i)}{\sqrt{\sum_{j=1}^{n} c_j^2 u^2(x_j)}} = \frac{|c_i|u(x_i)}{u(y)}$

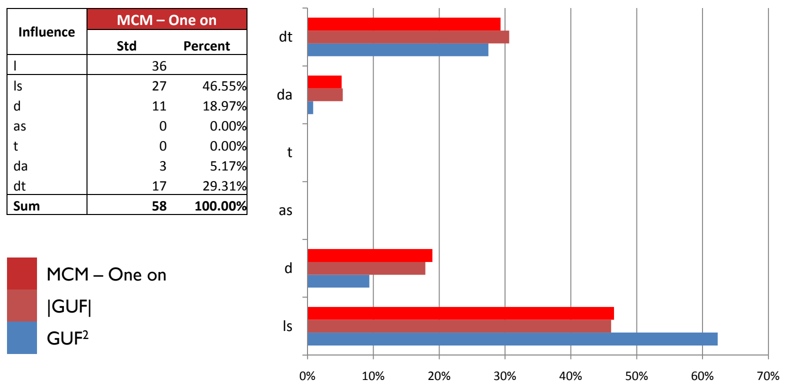
Influence	GUF abs	
	$\lvert c_i\rvert u(x_i)$	Percent
l	32	
ls	25	46.13%
d	9.7	17.90%
as	0	0.00%
t	0	0.00%
da	2.9	5.35%
dt	16.6	30.63%
Sum	**54.2**	**100.00%**



Beyond GUM-SP1

- Monte Carlo method
  - Turn all off except one influence

Influence	MCM – One on	
	Std	Percent
l	36	
ls	27	46.55%
d	11	18.97%
as	0	0.00%
t	0	0.00%
da	3	5.17%
dt	17	29.31%
Sum	**58**	**100.00%**



Beyond GUM-SP1

- Monte Carlo method
  - Turn one influence quantity off

Influence	MCM - One off		
	Std	Difference	Percent
l	36		
ls	24	12	33.33%
d	34	2	5.56%
as	36	0	0.00%
t	34	2	5.56%
da	34	2	5.56%
dt	31	5	13.89%

MCM - One off
MCM - One on
|GUF|
GUF²

Important:

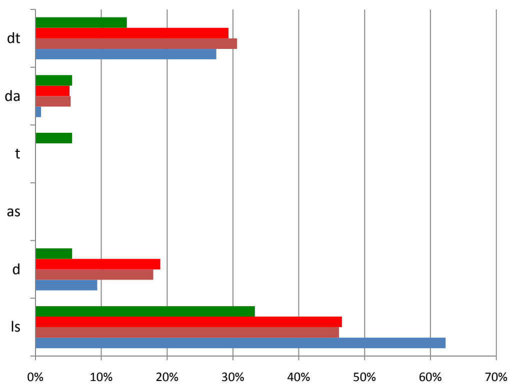
- Sum of the der percentage contributions is *not* 100%
- Direct comparison with the other methods only possible to a limited extent
- Percentage illustrates the *maximal reduction* of the measurement uncertainty through optimizing this influence

Beyond GUM-SP1

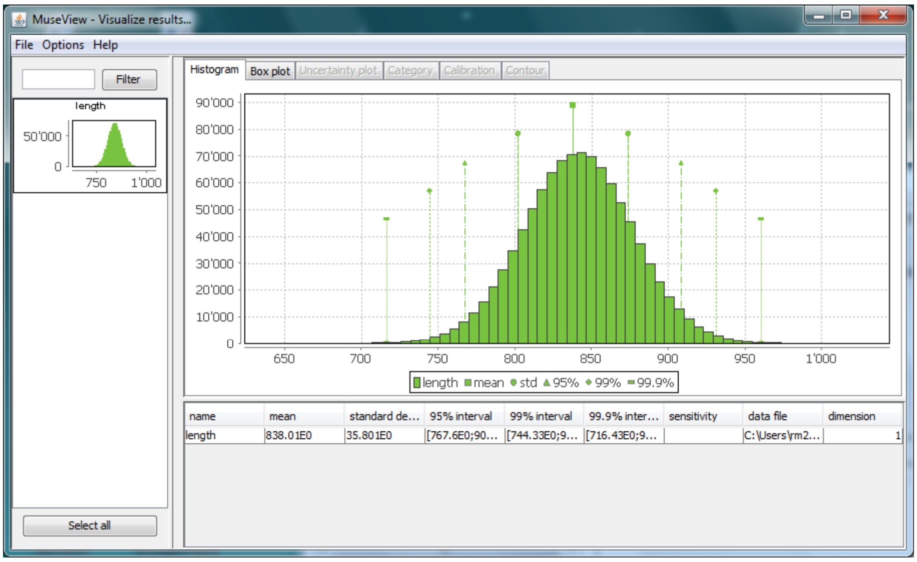
- Monte Carlo method
  - Turn one influence quantity off

Influence	MCM – One off		
	Std	Difference	Percent
l	36		
ls	24	12	33.33%
d	34	2	5.56%
as	36	0	0.00%
t	34	2	5.56%
da	34	2	5.56%
dt	31	5	13.89%

MCM – One off
MC – One on
|GUF|
GUF²



MuseLite – Basic User Interface



Thank You Very Much

- Simulation framework MUSE
  - Concepts of GS1 and additional new approaches realized in one framework
  - Entire implementation in C/C++
  - Robust long-run behavior (memory usage & computational time)
    (MUSE: Computational aspects of a GUM-supplement 1 implementation, Metrologia, 45, p586-593, 2008)
- Structured Modeling in XML
  - Stored in a pure text file
  - Open and easy complemental format
  - Strict language definition in XML
- Beta version available on:
  http://sourceforge.net/projects/freemuse/files/