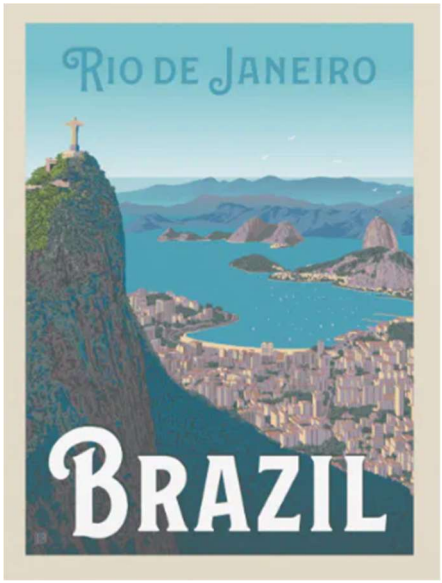
Should Gabriel visit Maria?

A short introduction to Bayes' Theorem



Maria and Gabriel are old friends from Porto Seguro, a small island in the middle of the Atlantic Ocean. Maria misses Gabriel, who joined the Navy for a long trip on the Sergas school ship. Maria is happy to have received a postcard from Gabriel, posted from Rio de Janeiro, informing her that he would visit her when the Sergas moored at Porto Seguro for three days on its way to Lisbon.

Oh no! COVID-19 outbreaks in Porto Seguro and on the Sergas, forcing both Maria and Gabriel to be tested for the disease. Both friends used the same rapid test, COVRAPID, and tested positive! The outbreak is stronger on the ship than on the island, with an estimated percentage of infected crew members and island population being 7 % and 1 %, respectively.

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Gabriel sent an email to Maria saying that they could meet because they had both received positive results from the COVRAPID test, so the chance of them both actually having the disease should be the same.

Maria had just studied Bayes' theorem, which states that the probability is related to the infection prevalence as well as the COVRAPID test performance (true positive rate, *TP*, or sensitivity, and false positive rate, *FP*). Unfortunately, COVID-19 is much more common on the ship than on the island.

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Maria sent the Eurachem/CITAC Guide on Qualitative Analysis* to Gabriel to help him to understand how prevalence and COVID test performance contribute to the infection probability:

$$PP(\mathbf{Maria}) = \frac{0.01 \cdot TP}{0.01 \cdot TP + (1 - 0.01) \cdot FP} = 0.620 = 62.0\ \%$$

$$PP(\mathbf{Gabriel}) = \frac{0.07 \cdot TP}{0.07 \cdot TP + (1 - 0.07) \cdot FP} = 0.924 = 92.4\ \%$$

where *PP*(Maria) and *PP*(Gabriel) are the probabilities of Maria and Gabriel being infected, given a *TP* of 81 % and a *FP* of 0.5 %. In Bayes' theorem these probabilities are called "posterior probabilities" because they result from an update of the initial information, based on COVID-19 prevalence, with the new information from the positive test.

After doing the calculation, Gabriel does not feel comfortable visiting Maria; Gabriel appears very likely to have the disease and a visit would increase the chance of Maria becoming infected if her test result is a false positive. The friends hope probabilities will be on their side when a new opportunity to meet arrives.

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Thomas Bayes (1701–1761) was an English Presbyterian minister and statistician who is known for formulating a theorem that describes how the probability of an event (such as infection) changes with new information: Bayes' theorem.

Reference

R Bettencourt da Silva and S L R Ellison (eds.) Eurachem/CITAC Guide: Assessment of performance and uncertainty in qualitative chemical analysis. First Edition, Eurachem 2021. Available from https://www.eurachem.org.